DISSECTION OF THE DODECAHEDRON

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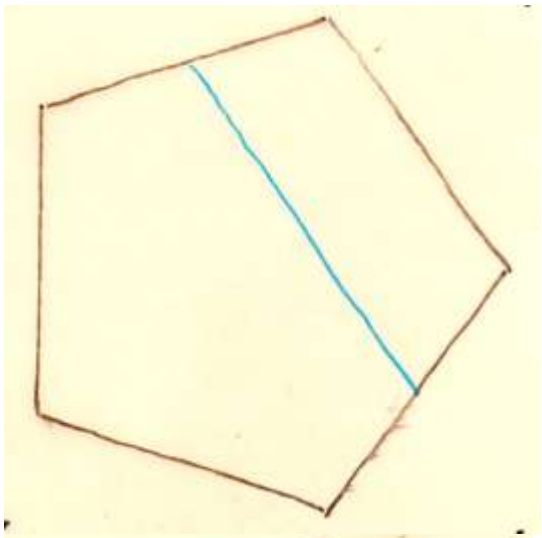
START

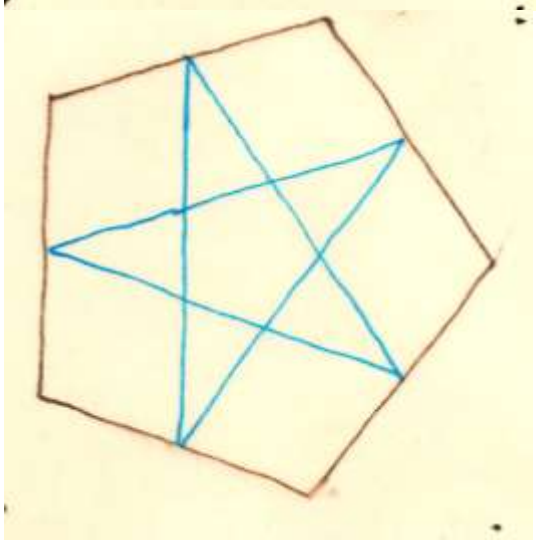
Let's start from the very beginning,
a very good place to start,
is a regular five sided figure called a pentagon,

'Regular' means that all the sides and internal angles of the pentagon are equal.

If we join the mid-points of two non-adjacent sides, we get a 'chord' which is parallel to the side in between the two bisected sides.

If we draw all the five chords, we get a beautiful five-pointed star (pentagram) inscribed within the original pentagon. Thus the pentagon gets dissected into 11 parts (facets) of 3 different types.





(A) One core pentagon
(B) Five rhombuses
(C) Five isosceles triangles

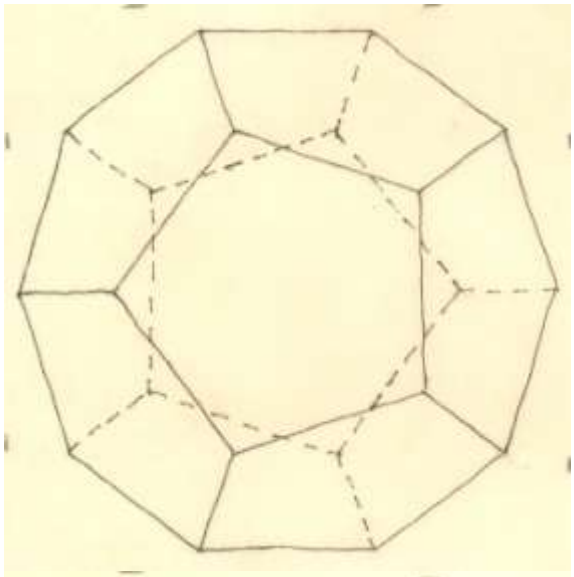
All figures in this presentation are free-hand sketches and so, may not be mathematically accurate, or to scale. The side of the pentagon is considered to be of unit length. You may choose whatever unit suits you best. All other lengths are expressed as multiples (or fractions) of the unit. It may be of interest to know that the 'Golden Ratio' (and functions of the same) plays a prominent role in defining the several lengths in these figures. The Golden Ratio is usually represented by the Greek letter 'Φ' and its value is $(\sqrt{5} + 1)/2$ or $2\text{Cos}(36°)$ which approximates to 1.618.... And just by way of an example, the length of the 'chord' is equal to $(\Phi + 1)/2$. The three types of figures (facets) are:

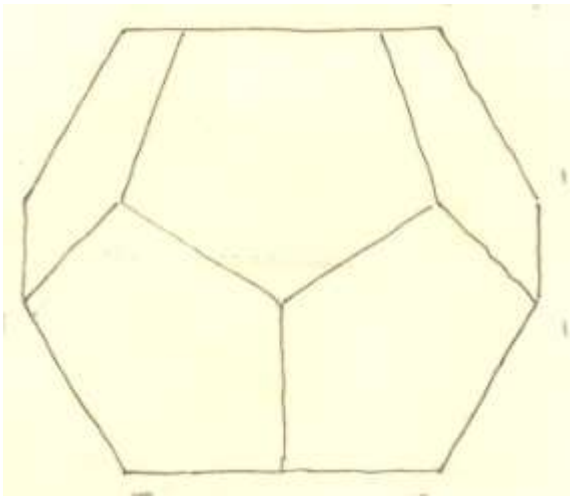
(A) A core pentagon (regular) whose sides are $(\Phi - 1)/2$ in length.
(B) Five rhombuses all of whose sides are ½ a unit in length and having two angles of 108° opposite each other, and two opposite angles of 72°.
(C) Five isosceles triangles whose short side is $(\Phi - 1)/2$ and the two long sides are ½ units long, having an included angle of 36°.

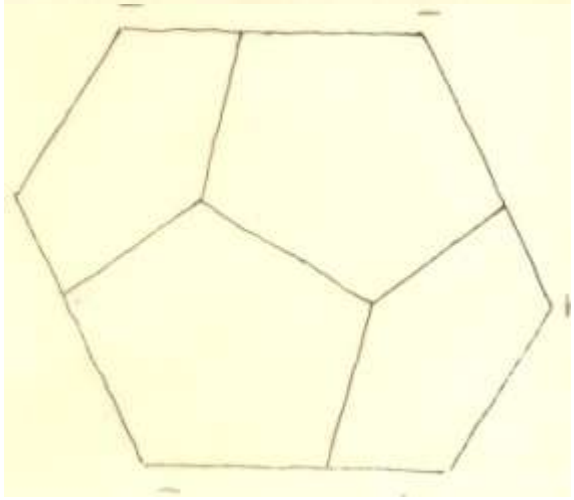
Besides these three types, there is a fourth type of figure (Type D) which we will soon encounter.

ENTERING THE THIRD DIMENSION

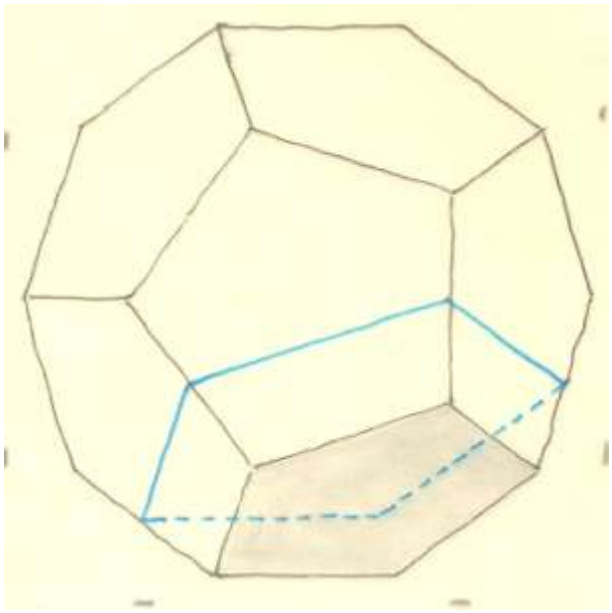
We now move one step up, into the third dimension. Twelve regular pentagons can form the surface of a ‘solid’ figure. It is called a Dodecahedron, and is one of the five regular Platonic Solids. When viewed from different angles -- top, side and front this is what it looks like.



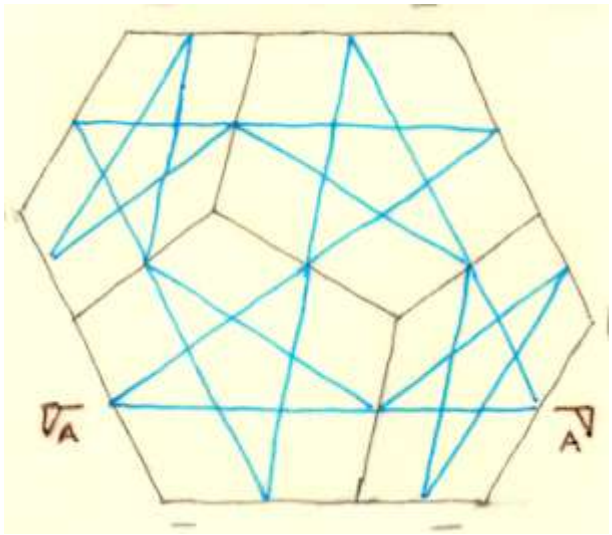




Consider any one pentagonal face. There are five edges radiating out at an angle from each of its five corners. If we join the midpoints of the five radiating edges, we get five 'chords' on each of the five faces adjacent to the face under consideration. And they all lie in one plane, parallel to that face. We can sort of create a 'slice' by cutting along that plane, the slice being in the shape of a frustum of a pentagonal pyramid.

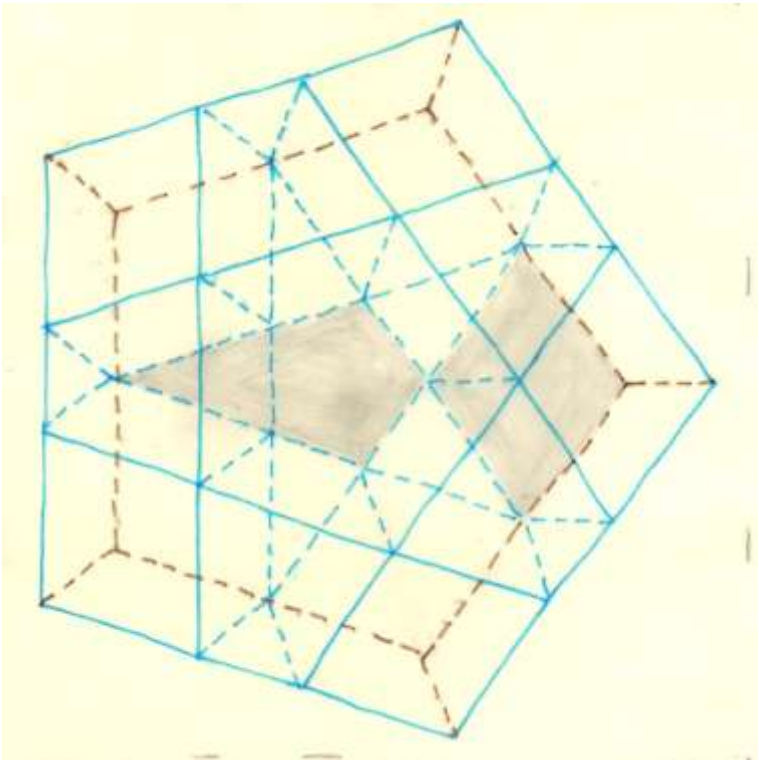


If we 'slice' off all the twelve faces of the dodecahedron, we get a real good 'sliced' and 'diced' dodecahedron! That is because all the twelve slices are not parallel to each other.



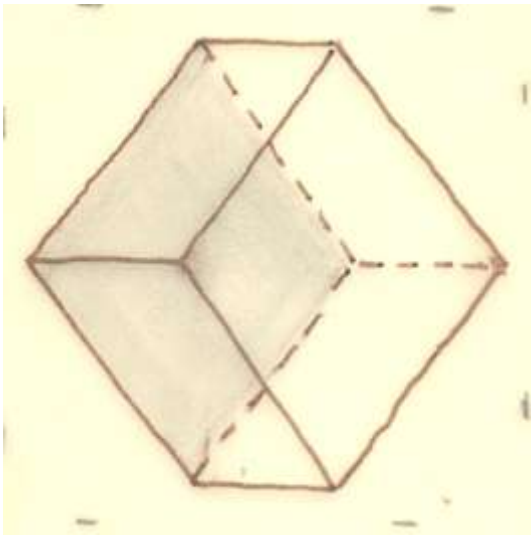
As there are twelve faces there will be twelve 'slices', all identical to each other. There will be some 'dices' common to more than one face (slice). In addition, there will be central core which does not belong to any of the slices. This core piece is a small regular dodecahedron. Since all the slices are identical, we will take up one slice – the bottom-most slice (marked A-A) for detailed inspection.

ANALYZING THE 'SLICE'



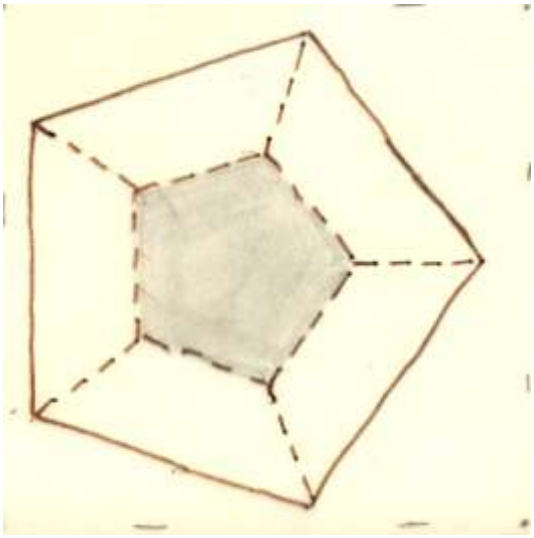
This is the lower-most slice viewed from the top (view A-A). It has eleven 'dices' of three types. Each of these dices has a facet on the bottom-most face of the dodecahedron (seen as a brown dotted pentagon). The base facet of three of these dices (one for each type) has been shaded for easy identification. The three different types of dices are named (1) the corner piece (2) the face piece and (3) the edge piece.

THE CORNER PIECE



This is the simplest piece. There is a corner piece at each of the twenty corners of the dodecahedron. Each corner piece 'lends' a facet to three of the faces of the dodecahedron meeting at that corner.
It consists of six rhombuses all congruent to each other. All the edges (twelve of them) are equal in length although in the figures above, they do not appear to be so, on account of the viewing angle. Actually, all facets are of Type (B).

THE FACE PIECE

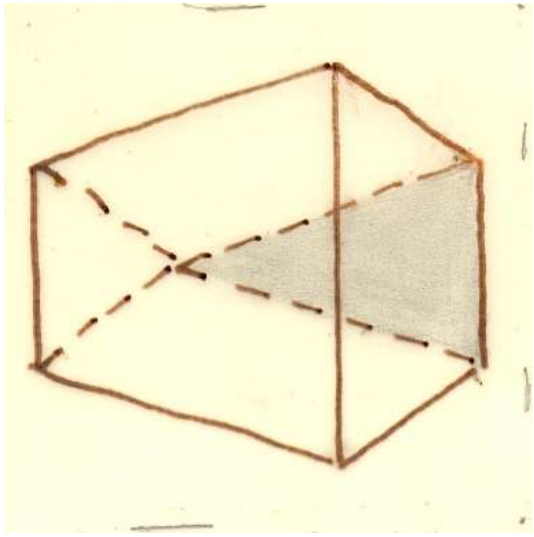


There are twelve such pieces, one for each face of the dodecahedron. It has seven facets – two pentagons of Type (A) (one big and one small) and five trapeziums. This is a fresh type of facet not encountered earlier, and is designated as Type (D).

Type (D) trapezium has two parallel sides of lengths (Φ-1)/2 and (Φ -1) and the other two sides are each ½ a unit in length. The angles at the ends of the short side are 108° and those at the end of the long side are 72°.

The sides of the larger pentagon are all (Φ -1) units long, i.e. twice the length of the smaller pentagon.

THE EDGE PIECE



To call this an edge piece is somewhat of a misnomer, as this piece has only one point located at the mid-point of an edge of the dodecahedron. There are thirty such pieces, one for each of the thirty edges of the dodecahedron.

It consists of six facets – two rhombuses of Type (B), two isosceles triangles of Type (C), and two trapeziums of Type (D). It 'lends' the two of its triangular facets to two of the adjacent faces of the dodecahedron.

AND FINALLY, THE CORE PIECE

The core piece is a piece that does not belong to any of the twelve slices, and yet is snugly fitted within all the twelve slices. There is only one such dice. None of its facets are exposed to the outside surface of the dodecahedron. It itself is a small dodecahedron having twelve regular pentagonal facets.

SUMMARY OF THE DISSECTION

This completes the dissection of the dodecahedron. In summary, there are:

- Twenty Corner Pieces
- Twelve Face Pieces
- Thirty Edge Pieces
- One Core Piece

making a total of sixty-three dices in all.